Event-triggered control for improving the positioning accuracy of connected vehicles equipped with DSRC

Adriana Simona Mihăiţă * **Chen Cai** * **Fang Chen** *

* *DATA61—CSIRO, 13 Garden St., Eveleigh, 2015, NSW, Australia*

Abstract:
Vehicle-to-Vehicle communication and Dedicated Short Range Communication systems have gained an increasing popularity in building vehicular applications for improving road safety, but the high level of positioning accuracy at the centimetre level is still far from being achieved. Various outages in transmitting the positioning information between neighbouring vehicles and errors in broadcasting their current locations can lead to a fail in generating accurate collision alerts that would help improve road safety.
The goal of this paper is to propose a modelling framework for applying an event-triggered control when the location transmitted by connected vehicles equipped with DSRC is lost due to unforeseen events. Firstly, we model the evolution of the DSRC transmitted positioning as a multi-state stochastic switching system by taking into consideration the distance from the transmitted location to the road center. A control interval is defined for the evolution of the positioning signal by using the road width to define the boundaries. Secondly, we propose an analytic method for determining the exit probabilities from the control interval, with the scope of anticipating any position anomalies and help applying the event triggered control when the control boundaries have been reached. Thirdly, we apply a cooperative location estimation method for improving the broadcast position information by using the accumulating trajectory segments from the moment of the anomaly alert.

Keywords: DSRC, connected-vehicles, positioning, event-triggered control, Markov Chains.

1. INTRODUCTION

Addressing congestion and avoiding traffic collisions has become the main focus of intelligent transportation systems (ITS) due to ever increasing number of road traffic deaths per annum (WHO, 2013). In order to improve road safety, various ITS strategic plans are already engaged in using advanced transport technologies such as Vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications (Grace et al., 2012).

Dedicated Short-Range Communication (DSRC) systems have recently known an increasing popularity due to their potential to enhance mobility safety and environmental security (Zeng et al., 2012). United States of America, Europe and Japan have already adopted the IEEE 802.11p standard and established dedicated bandwidths for DSRC (Li, 2010). Various projects and test bed initiatives have concentrated on: 1) analysing the effective communication range between two connected vehicles (DOT, 2011), 2) testing the probability of successful message reception (Jiang et al., 2006), measuring collision timings (Tang and Yip, 2010) or investigating signal priority for connected vehicles at signalized intersections (Gende et al., 2016). Despite a high reliability shown by the DSRC equipped systems, in 2014, the National Highway Transportation Safety Administration (NHTSA) assessed the need to further investigate DSRC related problems before releasing a deployment-level V2V communication system mandate (Harding et al., 2014).

The biggest challenge when using DSRC systems for connected vehicle (CV) applications is the accuracy of the vehicle position broadcasting through Basic Safety Messages (BSMs) to neighbouring vehicles. Many connected vehicle applications need sub-meter accuracy at the lane level, especially for real-time situational awareness (Alam et al., 2012). Bridging the gap between positioning accuracy and the necessary availability for CV applications represents an important challenge still to be tackled. Various works concentrated on improving the GPS positioning of DSRC-equipped connected vehicles by either proposing a Bayesian approach which uses received signal strength data from roadside equipment (RSE) to update the vehicle position (Jiangchen et al., 2016), or integrating together positioning and navigation map data which contains metadata for travellers (Efatmaneshnik et al., 2011; Van Hamme et al., 2013). While these methods are very promising, they are relying on the existence of additional equipment/features to be used (RSEe, digital maps), which represent supplementary costs for current test bed initiatives still trying to tackle the connected vehicle installation and usage.

Although in ideal operating conditions (clear sky, no obstructions), the DSRC equipped vehicles can present a good positioning accuracy, in more dense urban areas, tunnels or bridges, the positioning signal can suffer various outages and random perturbations (Alam and Dempster, 2013). As the number of connected vehicles on the roads will increase, there is true need to understand how various external events can influence the evolution of the transmitted positioning signal, and what are the most adapted control methods that can be applied for improving the positioning accuracy of DSRC systems in such conditions.

Due to a random evolution and transmission of DSRC location information, we believe that stochastic switching systems (SSS) are an appropriate tool to model the behaviour of such dynamical systems which are subject to random failures and sudden environmental disturbances. SSS are an important class of hybrid dynamical systems which contain a continuous part, usually represented by a family of subsystems driven by differential equations, and a discrete part, which can be a logical rule for switching between these subsystems. The idea of introducing the stochastic aspect in the hybrid systems has been initially presented in (Lygeros et al., 2008) and the modelling approach is now applied in robotics (Egerstedt and Hu, 2002), networking (Strelec et al., 2012), transportation systems (Pola et al., 2003), automotive systems (Balluchi et al., 2000) and biological systems (Kumar et al., 2013).

The continuous evolution of a SSS is interspersed by discrete events that can influence the switches between various states of the system. In the case of discrete-time systems, the control problem is easier to solve as the switching between the states of the systems take place at fixed sampling instants (Bemporad and Morari, 1999). But if the switches occur during the inter-sampling periods, this would lead to modelling errors and state-mismatch. In our case, although the BSMs are normally programmed to be sent out 10 times a second, if the DSRC positioning signal is lost due to random external events or the system has a failure to transmit the messages, then the constant timing between consecutive BSMs becomes stochastic. In order to avoid these errors, an appropriate solution would be to apply an *event-triggered control* (ETC) when specific abnormal switches occur in the evolution of the system or the transmitted position falls outside of the road. The main advantage of using ETC is the fact that it can be applied only when is needed, and is very efficient when the control methods are computationally expensive or when the energy consumption has specific limitations (Cogill et al., 2007; Åström and Bernahardsson, 2002). Applying a continuous procedure for re-estimating the car's position at every time step is time-consuming and would result in delays between communicating devices. Applying an adapted event-triggered control and anticipating when the system will suffer anomalies in positioning is a true challenge still to be tackled.

In this paper, we present the procedure and framework for modelling the evolution of DSRC transmitted positioning signal as a multi-state stochastic switching system. The main objective is to propose an event-triggered method for improving the positioning accuracy of connected vehicles equipped with DSRC when road side units do not exist and the dead-reckoning feature is failing to update the position of the vehicle. Section 2 introduces the context of our work, the motivation for this study as well as the main challenges we are trying to address. Section 3 presents the model description, the definition of anomalous events that can trigger the control as well as the framework for applying the ETC. For modelling the transitions between the states of the system we use continuous-time Markov chains (CTMC) with finite state space. In Section 3.2 we elaborate an analytical method for computing the exit probabilities from the control interval while in Section 4 we propose a collaborative positioning estimation method to be applied during ETC. Conclusions and future perspectives of this work are provided in Section 5.

2. MOTIVATION FOR THIS WORK

2.1 Context

The Cooperative Intelligent Transport Initiative (CITI) is a project currently undertaken by Transport for New South Wales (TfNSW) in partnership with DATA61 and the Australian Federal Government's Heavy Vehicle Safety Productivity Program (TfNSW, 2016), with the aim of building Australia's first semi-permanent DSRC test-bed over an area of 917 km^2 in the Illawarra Region of NSW (Tyler et al., 2016). Currently, almost 60 heavy vehicles, three signalised intersections and one roadside location have been equipped with DSRC units. According to TfNSW, 18% of the traffic on an important road in CITI consists of heavy vehicles, which are involved in 63% of fatal crashes (RMS, 2016).

In order to ensure road safety, the main challenge of the project is to address the generation of false collision alerts that hinders driving conditions and might result in drivers ignoring or not trusting the DSRC on-board warning device. Therefore, the first step was to investigate the accuracy of the transmitted positions between heavy vehicles, as reported from Basic Safety Messages (BSMs). The major finding indicated that 37.89% of transmitted BSMs during a specific period of time were incomplete or empty, while some vehicles presented various anomalies in positioning (e.g. one of the most active vehicles presented almost $\simeq$ 9% of anomalies). This work is therefore a continuation of previous results presented in (Mihaita et al., 2017) with the purpose of proposing a theoretical framework for detecting anomalous events and propose an adapted control strategy that would mitigate this risk.

2.2 Challenges to address

Although BSMs are normally transmitted 10 times a second, random outages of the system or failures in sending out the updated information can induce larger time intervals between BSMs and inaccuracies in the broadcast information. The positioning information in these messages is based on GPS measurements and in the case of brief loss of the GPS signal the position may be extrapolated from last known data in a process known as "dead reckoning" (Bento et al., 2012). However, in CITI, no additional sensors are connected to the DSRC unit and dead reckoning is restricted to interpolations from last known GPS locations. For the remainder of this paper, reference to "GPS" is actually a reference to "GPS-based positioning information" as is transmitted by a vehicle in a BSM. Therefore, the accuracy of the "transmitted GPS positioning" is not independent of the DSRC unit, but is a mix of processing and transmission methods. Modelling the stochastic behaviour of such systems represents a true challenge, especially for finding an adapted method for re-estimating the positioning information when no road side units are available or the dead-reckoning functionality is not accurate. As well, we seek to anticipate when the positioning of the vehicle would fall outside of the road width and how to improve the location estimation of the vehicles in this situation.

3. MODEL DESCRIPTION

In the following, we denote by N_G the total number of DSRC GPS observations transmitted in a single BSM. A GPS observation is described by its location $A_i = (L_i, l_i)$, $i \in \{1, ..N_G\}$ registered at time t_i, where L_i and l_i denote the longitude, and the latitude respectively. Let Δt be the time between two consecutive GPS observations; in this study we consider Δt to be stochastic due to possible random loss of the transmission signal. Each GPS observation A_i can be mapped to a specific road segment g_j, $j \in \{1, G\}$, with the same spatio-temporal characteristics. A symbolic graphical representation of three consecutive GPS observations over a selected road section is provided in Figure 1. The road section has two lanes of width L in the same direction and, in this case, the line separating the two lanes represents the road center used for computing the noise. Let N_i be the distance (or noise) between a registered GPS location and its projection on the road centre line at a specific time t_i. For simplicity, in the rest of this paper, we will use the notion of noise when referring to the distance between a GPS observation and the road center.

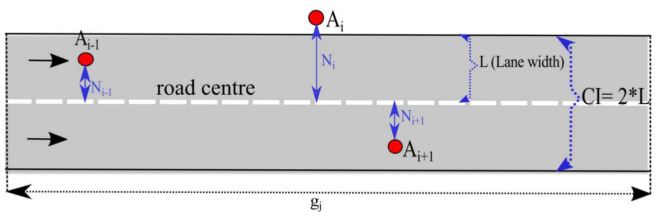

Fig. 1. Example of 3 GPS observations.

Let $s \in S$ be a discrete state of the system corresponding to a specific GPS observation, $S = \{1, ..N_G\}$ a finite state space and $x(t)$ the state variable denoting the continuous evolution of noise registered in each state of the system. The system remains in a specific state s until a random switch occurs; it will then transition to a new state described by a new noise level. We model this behaviour as a stochastic switching system described by:

$$\begin{cases} \dot{x}(t) = N_{A(t)} \\ x(0) = x_0 \end{cases} \tag{1}$$

where $x_0 \in \mathbb{R}$ is the initial state of the system, $A(t)$ the continuous-time Markov chain defined on S, and $N_{A(t)}$ the accumulated noise when the system is in a particular state of the Markov chain. The hybrid behaviour of the system is described by the continuous evolution in time of the state variable $x(t)$, while the system randomly switches from one state to another, as described by the associated CTMC. Let Q be the transition matrix of the system:

$$\mathbf{Q} = \begin{pmatrix} -\sum_{j\neq 1}\lambda_{1,j} & \dots & \lambda_{1,N} \\ \lambda_{N,1} & \dots & -\sum_{j\neq N}\lambda_{N,j} \end{pmatrix} \tag{2}$$

where $\lambda_{i,j}$ is the transition rate from one state i to another state j. Consequently, the transition probability between i and j is $p_{i,j} = \lambda_{i,j}/\sum_{j\neq i}\lambda_{i,j}$.

As mentioned earlier, the system can suffer random failures in transmitting or updating the information regarding its location. Figure 2a) shows a short sequence of real DSRC GPS observations as transmitted by a connected heavy vehicle operating in CITI. The road center is a Google Map shape file which is used for computing the noise as a Vicenty distance (Vicenty, 1975) between the DSRC GPS observations and the corresponding mapped point on a road segment. This process is known as a Map-Matching (MM) procedure. MM algorithms can integrate GPS positioning data with spatial road network data in order to identify the correct link on which a vehicle is travelling and the exact location of a vehicle on that link (Greenfield, 2002). While MM methods have received a lot of attention in the literature (Hashemi and Karimi, 2014; Miwa et al., 2012), our main objective in this paper is to focus on event-triggered control techniques that can be applied when special events or anomalies may appear.

Figure 2b) shows the corresponding noise computed for each GPS observation plotted in Figure 2a). As the official lane width in Australia is $L = 3.5m$, we define this limit as the Upper Control Limit (UCL). Any noise below this limit ($N_i \in [0, 3.5m]$) is considered as normal. Noise levels that fall outside UCL ($N_i \geq 3.5m$) are considered anomalies and will trigger the control for re-estimating the DSRC transmitted location. We therefore define the following events that can change the evolution of the system:

- *Normal switching events*: which are independent of the control limits, and indicate a normal switch between the states of the system.
- *Anomalous events*: which appear on random occasions and indicate that the positioning signal is erroneous and falls outside of the normal road width. These events trigger a special type of control that needs to be applied in order to re-estimate the transmitted positioning of the vehicle, which we denote as *Event-Triggered Control* (ETC).

We therefore define the control interval ($CI = 2L$) as the interval in which normal noise levels may appear and UCL and LCL as the upper, respectively lower control limits of the SSS considered in this paper. The control process is stochastic with the objective of maintaining $x(t) \in CI$. While other control techniques can be imagined for this problem (PID, anti-windup, Lyapunov, etc.), our main objective is to propose a control method which is based on a stochastic process and that could be further used to minimize the energy to maintain the positioning of the vehicle inside the road width. For simplicity, we consider the road width to be fixed while the switches between the states to be stochastic. The current method is adapted from state-feedback control in a simplified stochastic version. The method is similar to the original approach formu-

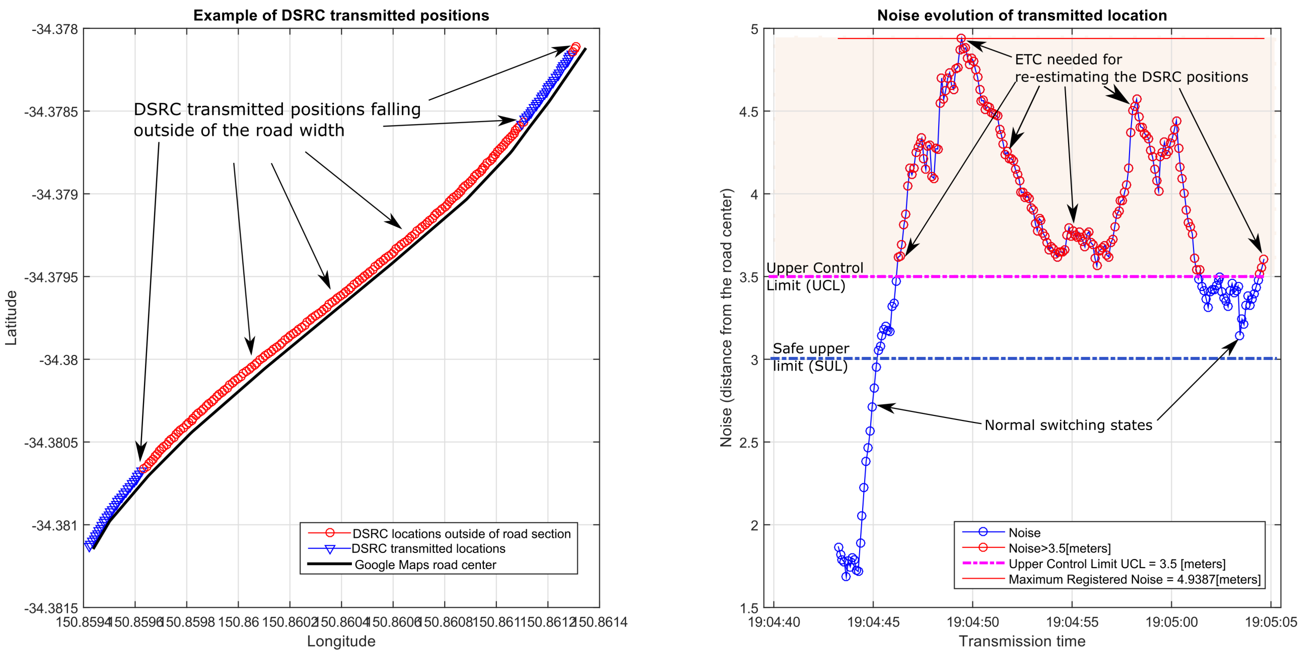

Fig. 2. a) DSRC transmitted positions along the road center from Google Maps. b) Evolution of noise along the road segment falling outside the Upper Control Limit UCL.

lated by (Åström and Bernahardsson, 2002) and represents our original inspiration for this study. The event-triggered controlled system is described by:

$$\begin{cases} \dot{x}(t) = N_{A(t)} + u_{A(t)}(t) + w(t) \\ x(0) = x_0 \end{cases} \tag{3}$$

where $u_{A(t)}(t)$ is the control command applied when the upper or lower control boundaries have been reached and $w(t)$ is a zero-mean white Gaussian noise related to the control command. We can therefore express $u_{A(t)(t)}$ as:

$$u_{Z(t)}(x(t)) = \begin{cases} 0, & \text{if } x(t) \in CI \\ \Delta N(t), & \text{if } x(t) \notin CI \text{ zone}, \forall i \in S \end{cases} \tag{4}$$

where $\Delta N(t)$ represents the difference between the estimated noise and current noise in order to re-adjust the positioning of the vehicle so that the noise level remains inside the control interval. The main challenge is therefore to be able to find an event-triggered control command which is adapted to the stochastic evolution of the system with random anomaly events and can improve the system performance.

3.1 Framework

Applying the control only when is needed has a great advantage of minimizing the effort of a continuous repositioning of the vehicle which can be time and energy consuming. Recent studies have investigated the possibility of using cooperative positioning (CP) methods for re-estimating the transmitted location of a connected vehicle by combining position-related data from neighbouring vehicles. While conventional CP systems (differential GPS, real-time kinematic GPS, assisted GPS) may suffer from limitations such as low signal coverage, weak signals or accuracy (Hofmann-Wellenhof et al., 2001), modern CP methods have gained a lot of attention as they are using V2V and V2I connections to estimate the position of a vehicle when the number satellites is small or GNSS is unavailable (Tan, 2010). DSRC systems can provide range data as well as heading, speed and acceleration of vehicles, which enables more complicated techniques that have potential to enhance position accuracy, such as Bayesian filtering (Liu et al., 2015) or Kalman filter (Efatmaneshnik et al., 2011).

But anticipating when to apply the re-estimation of the positioning information which will be transmitted remains a challenge for which we propose the analytical solution presented in Section 3.2. In Figure 3 we propose the general framework for applying the ETC, which can represent a baseline for further tests and studies of positioning methods. By using information broadcast by the DSRC unit with dead-reckoning feature (Step 1), we apply a Map-Matching procedure (Step 2) for computing the distance (noise) between the DSRC GPS observations and the road center. We then model the noise evolution at every time step using a SSS with a CTMC which helps us to compute the exit probabilities towards the control boundaries (Step 3). If these probabilities reach a certain predefined threshold P (e.g. $P = 80\%$), then the ETC can be applied in advance in order to re-estimate the positioning of the vehicle from the associated road segment (Step 4). The ETC can be applied for each state of the system until the exit probabilities fall below the specified threshold.

3.2 Exit probabilities

The analytic method we propose in this section is inspired from the studies of (Gardiner, 2004), for computing the mean exit times that a particle needs to exit a control zone with absorbing barriers. Another interesting analytic method for determining the maximum level and the hitting

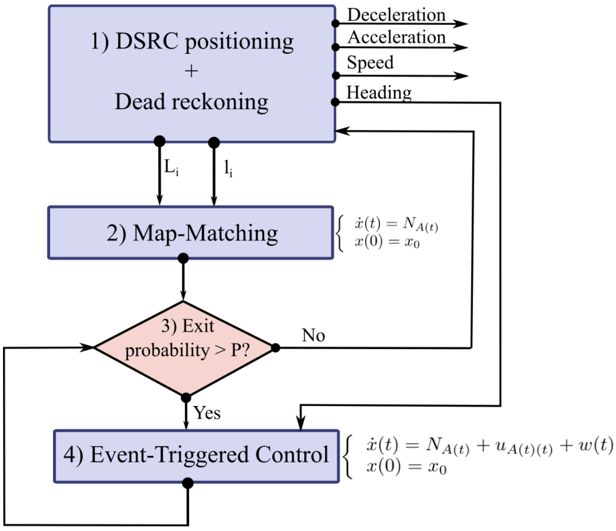

Fig. 3. ETC framework.

probabilities for stochastic fluid flows is presented in (Sericola and Remiche, 2011), with the use of matrix differential Riccati equations. We define the exit probabilities in the following.

Definition 3.1. Let $\{A_n\}_{n\in\mathbb{N}_+}$ be a continuous-time Markov chain and B a subset of the state space taking values inside the control interval CI. We define the exit probability $\pi_e^j(x)$, as the probability for the system to exit B, when starting in a state j:

$$\pi_e^j(x) = Prob\{x \notin B | x(0) = x_0, A(0) = j\}.$$

Considering the multi-state stochastic switching system representing the evolution of noise in time, we use backward Kolmogorov equations, for computing the exit probabilities towards the control limits (Confortola and Fuhrman, 2013). In this section, we present only the case for the exit towards UCL, as the exit towards LCL is similar. Let us consider the following differential equation:

$$\mathbf{R} \cdot \frac{d\boldsymbol{\pi}_e(x)}{dx} + \mathbf{Q}^T \cdot \boldsymbol{\pi}_e(x) = 0 \tag{5}$$

where $\mathbf{Q}^T$ is the transposed transition matrix of CTMC defined in (2), $\mathbf{R}$ is a diagonal matrix of the noise evolution associated to the states of the system:

$$\mathbf{R} = \begin{pmatrix} N_1 & \dots & 0 \\ 0 & \dots & N_{N_G} \end{pmatrix} \tag{6}$$

and $\boldsymbol{\pi}_e(x)$ is a probability column vector defined as:

$$\boldsymbol{\pi}_e(x) = \left[\, \pi_e^1(x) \;\; \pi_e^2(x) \;\; \dots \;\; \pi_e^{N_G}(x) \,\right]^T$$

with $\pi_e^j(x)$ as the *probability to reach* UCL *when starting from* x *in state* j. Therefore, we search for the exit probabilities towards the upper control limit :

$$\pi_e^j(x) = Prob\{x \geq UCL | x(0) = x_0, A(0) = j\}$$

which respect the boundary conditions: $\{\pi_e^j(UCL) = 1$ if $N_j \geq UCL\}$ and $\{\pi_e^j(LCL) = 0$ if $N_j \geq LCL\}$. In other words, when the system is already at the upper control limit, the probability to to exit towards UCL is 1, indicating an urgent need to apply the ETC. Similarly, if the system would be at the LCL, the probability to exit towards UCL would be 0. The vice-versa is valid for the exit probability towards LCL. Solving the backward Kolmogorov equations (5) for the exit probabilities, with the previous boundary conditions, provides the following solutions:

$$\boldsymbol{\pi}_e(x) = e^{-\mathbf{R}^{-1}\cdot\mathbf{Q}^T\cdot(x-LCL)} \cdot \boldsymbol{\pi}_e(LCL) \tag{7}$$

which can be computed at every switch between the states of the system and to decide whether the ETC should be applied or not. For best optimizing the application of ETC and to avoid delays in location re-estimation, one would need to set up a specific probability threshold P, as mentioned in the previous section. As an example, if the probability to hit the UCL is 80%, then the chances of a future anomaly are high, therefore the ETC could be triggered in advance. This event is classified as anomalous and helps preventing the delays in location re-estimation by anticipating when the system might reach the boundaries of the control interval.

4. EVENT-TRIGGERED POSITIONING ESTIMATION

Besides the information about the vehicle location, the DSRC unit broadcasts information about the acceleration, speed, elevation, brakes, and heading Θ (angle of the vehicle relative to the horizontal direction of movement). The heading and distance between DSRC locations can be used to re-estimate the vehicle's trajectory from the moment when an anomaly detection has been released. The following decentralized cooperative method is adapted from (Li et al., 2015) in which the authors applied it for estimating the location of mobile users in disruptive tolerant network, by accumulating walking segments.

Let $A_0 = (L_0, l_0)$ be the initial point from where the system has started to receive anomaly alarms ($\pi_e^0(x) \geq P$) and $A_c = (L_c, l_c)$ the location point where the system has reached (or even exceeded UCL) and has to re-estimate its current position. A transition movement from one location to another can then be approximated based on the distance between two consecutive locations of the same vehicle which we denote (D_i), as well as the heading of the vehicle. So the coordinates of estimated $\widehat{A_c}$ can be approximated by accumulating the past trajectory segments from the moment when the ETC alarm has been activated:

$$\widehat{L_c} = L_0 + \sum_{i=1}^{M} D_i sin\Theta_i \tag{8}$$

$$\widehat{l_c} = l_0 + \sum_{i=1}^{M} D_i cos\Theta_i \tag{9}$$

Theoretically, if we have access to the initial location when the alarm has been triggered, by using equations (8)-(9), we can then estimate its location as any given moment in time. But various errors can occur and accumulate during the transitions from one state to another (system failures, stochastic delays between consecutive locations), reason for which we strongly believe that the positions received from other connected vehicles along the road can help improving the positioning estimation in a collaborative process.

Consider the case of a DSRC connected vehicle with estimated positioning $\widehat{A_c}$ at UCL, which encounters another DSRC vehicle with location B_n. We then apply a two-step verification test: 1) if the Vicenty distance between the two vehicles is inferior to the DSRC range (which is normally set up at 250 meters according to (Efatmaneshnik et al., 2011)), then the second vehicle is considered to be a *valid* candidate with possible correct positioning 2) if the exit probability of the second vehicle exceeds as well the established probability threshold P or is already outside of the control interval ($\pi_e(N_{B_n}) \in [P, 1]$), then the second vehicle is considered to be anomalous as well and its location will not be used for improving the first car's location; otherwise, if the neighbouring vehicle has low exit probability nor hasn't reached the CI limits then it can be considered as an accurate neighbour and its position B_n can be used for further refinements of the first car's position.

In this case, the control command applied for adjusting the initial noise level of the first car, would be completed with the difference between the noise of the second car N_{B_n} and the noise of the first car N_{A_c} : $\Delta N(t) = N_{B_n} - N_{A_c}$ (see equation 3).

Final step is to adjust as well the trajectory of the system from A_0 location when the ETC system detected the anomaly, up to the last location B_n, whose coordinates now satisfy: $L_B = L_0 + \sum_{i=1}^{M} D_i^B sin\Theta_i$ and $l_B = l_0 + \sum_{i=1}^{M} D_i^B sin\Theta_i$. This would need further adjustments of all previous distances between consecutive transmitted DSRC locations, which can be solved as a linear programming optimisation problem. Lastly, one would need to verify that the error between the adjusted distances and the original ones remain under specific thresholds ε.

5. CONCLUSION

In this paper we presented an event-triggered control approach for improving the location estimation of DSRC transmitted positions in a connected vehicle environment which may be affected by unforeseen perturbations. The main advantage of the method is that the event-triggered control is applied only if anomalous events appear in the system evolution. By computing the exit probabilities towards the boundaries of the control interval one could verify what are the chances that the system location will deteriorate. This step is the starting point of searching for neighbouring vehicles which have correct DSRC location and respect the same control boundaries. We are currently focused on investigating various modern cooperative positioning methods which can be applied together with an ETC approach in a CV environment and which can improve the estimation of transmitted DSRC location. CITI is an ongoing project, with aims to investigate the use of DSRC for signalised intersections, as well as improving road safety especially in high concern public areas (schools, kindergardens). Ongoing work is also focusing on the quality of the GPS positioning on a a small number of light vehicles which are currently being equipped with both GPS and GLONASS for improving the location detection.

ACKNOWLEDGEMENTS

The authors of this work are grateful for the support provided by the Road for Safety and Technology from Transport for New South Wales, Australia.

REFERENCES

Alam, N., B., A.T., and Dempster, A. (2012). An instantaneous lane-level positoning using dsrc carrier frequency offset. *IEEE T. on Intelligent. Tr. Syst.*, 13(3).

Alam, N. and Dempster, A. (2013). Cooperative positioning for vehicular networks: Facts and future. *IEEE Transactions on Intelligent Transportation Systems*, 14(4), 1708–1717.

Åström, K. and Bernahardsson, B. (2002). Comparison of riemann and lebesque sampling for first order stochastic systems. *Proceedings of the* 41^{st} *IEEE Conference on Decision and Control.*

Balluchi, A., Benvenuti, L., Benedetto, M., Miconi, G., Pozzi, U., Villa, T., Wong-Toi, H., and Sangiovanni-Vicentelli, A. (2000). Maximal safe set computation for idle control of an automotive engine. In *Hybrid Systems Computation and Control*, volume 1790, 32–44. Springer.

Bemporad, A. and Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3), 407–427.

Bento, L.C., Parafita, R., and Nunes, U. (2012). Inter-vehicle sensor fusion for accurate vehicle localization supported by v2v and v2i communications. *15th International IEEE Conference on ITS*, 907–914.

Cogill, R., Lall, S., and Hespanha, J. (2007). A constant factor approximation algorithm for optimal estimation subject to communication costs. *Proceedings of the 2007 American Control Conference.*

Confortola, F. and Fuhrman, M. (2013). Backward stochastic differential equations associated to jump markov processes and applications. *arXiv:1302.0679 [math].*

DOT, U. (2011). Vehicle safety communications applications (vsc-a) final report, dot hs 811 492a. Technical report, Research and Innovative Technology Administration.

Efatmaneshnik, M., Kealy, A., Balaei, A., and Dempster, A.G. (2011). Information fusion for localization within vehicular networks. *Journal of Navigation*, 64(3), 401–416.

Egerstedt, M. and Hu, X. (2002). A hybrid control approach to action coordination for mobile robots. *Automatica*, 38(1), 125–130. doi:10.1016/S0005-1098(01)00185-6.

Gardiner, C. (2004). *Handbook of stochastic methods for physics, chemistry and the natural sciences.* Springer, Berlin, 3 edition.

Gende, M., Chowdhury, M., Dey, K., and Sarasua, W. (2016). Connected-vehicle technology for allowing priority requests at signalized intersections: Analysis. *Proceedings of the Transportation Research Board, 95th Annual Meeting.*

Grace, N., Oxley, C., Sloan, S., Tallon, A., Thornton, P., Black, T., and Easton, A.V. (2012). Transforming transportation through connectivity: Its strategic research plan, 2010/2014. Technical report, US DOT's Volpe Center.

Greenfield, J. (2002). Matching gps observations to locations on a digital map. *Proceedings, Transportation Research Board, 81st Annual meeting.*

Harding, J., Powell, G., Yoon, R., Fikentscher, R., Doyle, J., Sade, C., Lukuc, D., Simons, M., and Wang, J. (2014). Vehicle-to-vehicle communications: Readiness of v2v technology for application. Technical report, National Highway Traffic Safety Administration.

Hashemi, M. and Karimi, H.A. (2014). A critical review of real-time map-matching algorithms: Current issues and future directions. *Computers, Environment and Urban Systems*, 48, 153–165.

Hofmann-Wellenhof, B., Lichtenegger, H., and Collins, J. (2001). *Global Positioning System: Theory and Practice*. Springer-Verlag Wien.

Jiang, D., Taliwal, V., Meier, A., Holfelder, W., and Herrtwich, R. (2006). Design of 5.9 ghz dsrc-based vehicular safety communication. *Journal of Wireless Communications*, 13(5), 36–43.

Jiangchen, L., Jie, G., Hui, Z., and Tony, Q. (2016). A rse-assisted gps-rss hybrid lane-level positioning system for connected vehicles. *Proceedings of Transportation Research Board 95th Annual Meeting.*

Kumar, G.P., Buffin, A., Pavlic, T.P., Pratt, S.C., and Berman, S.M. (2013). A stochastic hybrid system model of collective transport in the desert ant *Aphaenogaster cockerelli*. In *Proceedings of the 16th ACM International Conference on Hybrid Systems: Computation and Control*, 119–124. Philadelphia, PA.

Li, W., Hu, Y., Fu, X., Lu, S., and Chen, D. (2015). Cooperative positioning and tracking in disruption tolerant networks. *IEEE Transactions on Parallel and Distributed Systems*, 26(2), 382–391. doi: 10.1109/TPDS.2014.2310471.

Li, Y. (2010). An overview of the dsrc/wave technology. *Proceedings of the 7th International Conference on Heterogeneous Networking for Quality, Reliability, Security and Robustness, QShine, Houston, USA*, 544–558. doi: 978-3-642-29222-4-38.

Liu, J., g. Cai, B., h. Wen, Y., and Wang, J. (2015). Integrating dsrc and dead-reckoning for cooperative vehicle positioning under gnss-challenged vehicular environments. *International Journal of Ad Hoc and Ubiquitous Computing*, 19(1-2), 111–129.

Lygeros, J., Tomlin, C., and Sastry, S. (2008). *Hybrid systems: modeling, analysis and control*. Lecture Notes.

Mihaita, A.S., Tyler, P., Menon, A., Wen, T., Ou, Y., Chen, C., and Chen, F. (2017). An investigation of positioning accuracy transmitted by connected heavy vehicles using dsrc. *Proceedings of Transportation Research Board 96th Annual Meeting, Washington D.C.*

Miwa, T., Kiuchi, D., Yamamoto, T., and Morikawa, T. (2012). Development of map matching algorithm for low frequency probe data. *Transportation Research Part C: Emerging Technologies*, 22, 132–145.

Pola, G., Bujorianu, M., Lygeros, J., and di Benedetto, M. (2003). Stochastic hybrid models: An overview. *Proceed. IFAC Conference on Analysis and Design of Hybrid Systems.*

RMS (2016). Picton road crash history. URL `http://tinyurl.com/h4kgmsg`.

Sericola, B. and Remiche, M.A. (2011). Maximum level and hitting probabilities in stochastic fluid flows using matrix differential riccati equations. *Methodology and Computing in Applied Probability*, 13(2), 307–328. doi: 10.1007/s11009-009-9149-z.

Strelec, M., Macek, K., and Abate, A. (2012). Modeling and simulation of a microgrid as a stochastic hybrid system. In *2012 3rd IEEE PES International Conference and Exhibition on Innovative Smart Grid Technologies (ISGT Europe)*, 1–9.

Tan, Y. (2010). Positioning techniques with two gnss satellites over time. *Proceedings of ION GNSS*, 208–216.

Tang, A. and Yip, A. (2010). Collision avoidance timing analysis of dsrc-based vehicles. *Accident Analysis and Prevention*, 42, 182–195.

TfNSW (2016). Cooperative intelligent transport initiative. URL `http://tinyurl.com/gmfu9v8`.

Tyler, P., Wall, J., and Vecovski, V. (2016). Citi-an update on australia's first pilot deployment of cits. *Proceedings of 23rd ITS World Congress, Melbourne.*

Van Hamme, D., Veelaert, P., and Philips, W. (2013). Lane identification based on robust visual odometry. *16th International IEEE Annual Conference in ITS, Hague.*

Vicenty, T. (1975). Direct and inverse solutions on the ellipsoid with application of nested equations. *Survey Review XXII*, 176, 88–93.

WHO (2013). *Global status report on road safety 2013: supporting a decade of action*. World Health Organization.

Zeng, X., Balke, K.N., and Songchitruksa, P. (2012). Potential connected vehicle applications to enhance mobility, safety, and environmental security. Technical report, Southwest Region University Transportation Center, Texas Transportation Institute, Texas A and M University System.